Number Bases

On Thursday we discussed the binary number system; however, I did not have anything prepared. And so I have prepared the following.

1) Decimal or Base 10.

Let's start with decimal.

a) Number of digits

The decimal system uses 10 digits: 0,1,2,3,4,5,6,7,8,9.

b) Base

The base for the decimal system is 10. So when we see a number like 9876.54, we interpret it using the following

Thousands	Hundreds	Tens	Units	Tenths	Hundredths
9	8	7	6	5	4

In fact we should use the following instead.

10^3	10^2	10^1	10^0	10^{-1}	10^{-2}
9	8	7	6	5	4

To give us its value

$9x10^3 + 8x10^2 + 7x10^1 + 6x10^0 + 5x10^{-1} + 4x10^{-2}$

$= 9000 + 800 + 70 + 6 + 0.5 + 0.04$

$= 9876.54$

This can be applied to any number base system.

2) Binary or base 2

   a) Number of digits is 2: 0,1.
   b) Base is 2.
   c) Example: what is 1010 binary in base 10?

2^3	2^2	2^1	2^0
1	0	1	0

To get its value in terms of the decimal system

$1x2^3 + 0x2^2 + 1x2^1 + 0x10^0$ = 1x8+0x4+1x2+0x1 =8+2=10

3) Octal or base 8

a) Number of digits is 8: 0,1,2,3,4,5,6,7.
b) Base is 8.
c) Example: what is 7654 octal (or base 8) in base 10.

8^3	8^2	8^1	8^0
7	6	5	4

To get its value in terms of the decimal system

$7x8^3 + 6x8^2 + 5x8^1 + 4x8^0$

$= 7x512+6x64+5x8+4x1$

$=4584 + 384 + 40 + 4$

$= 4012$

4) Hexadecimal or Base 16

a) Number of digits is 16 (and we have to invent some new numeric characters): 0,1,2,3,4,5,6,7,8,9, A, B, C, D, E, F where A stands for 10 decimal, B for 11, up to F for 15.
b) Base is 16.
c) Example: what is BA98 hexadecimal in base 10.

16^3	16^2	16^1	16^0
B	A	9	8

To get its value in terms of the decimal system

$Bx4096 + Ax16^2 + 9x16^1 + 8x16^0$

= 11x4096+10x256+9x16+8x1

=45056 + 2560 + 144 + 8

= 47768

5) Why do we need Hexadecimal? It's short of a concise shorthand.

Note that we can use 4 binary digits to represent a single hexadecimal digit.

Binary	Hexadecimal	Binary	Hexadecimal
0000	0	1000	8
0001	1	1001	9
0010	2	1010	A (or 10)
0011	3	1011	B (or 11)
0100	4	1100	C (or 12)
0101	5	1101	D (or 13)
0110	6	1110	E (or 14)
0111	7	1111	F (or 15)

Since computers use a basic 8-bit entity or a byte, we can use two hexadecimal digits.

Let's say we have a number, which is represented by 2 hexadecimal digits, FA.

Its value is

$Fx16^1 + Ax16^0$

$= 15x16 + 10x1$

$= 240 + 10$

$=250$

If binary was used then

11111010

$=1x2^7+1x2^6+1x2^5+1x2^4+1x2^3+0x2^2+1x2^1+0x2^0$

$=128+64+32+16+8+0+2+0$

$= 250.$

Which is easier to use?